Workshop in Arithmetic Geometry

September 23-24, 2023
Capital Normal University
Teaching Builing 2, Room 913

Schedule

Saturday, 23 of September

09:30–10:30	Jilong Tong (Capital Normal University)
Harder-Narasimhan stratifcation in p-adic Hodge theory	
10:40–11:40	Bin Zhao (Capital Normal University)
Refined spectral halo for eigencurves and p-adic slopes of crystabelline representations	
11:40–14:00	Lunch
14:00–15:00	Yongqi Liang (University of Science and Technology of China)
Hasse principle for odd genus hyperelliptic curves	
15:10–16:10	Yang Cao (University of Science and Technology of China)
Some new descent methods to weak approximation	
16:20–17:20	Dasheng Wei (Chinese Academy of Sciences)
Strong approximation of the singular Del Pezzo surface	

Sunday, 24 of September

08:30–09:30	Yong Hu (Southern University of Science and Technology)
Strong approximation and integral quadratic forms over affine curves	
09:40–10:40	Fei Xu (Capital Normal University)
Geometric Springer theorem for quadratic bundles over smooth affine curves under finite coverings of odd degree	
10:50–11:50	Zhiyu Tian (Peking University)
Local-global principle for zero cycles over global function fields	
11:50–14:00	Lunch
14:00–15:00	Jinbo Ren (Xiamen University)
Bounded Generation: a diophantine approximation approach	
15:10–16:10	Zhizhong Huang (Chinese Academy of Sciences)
Revisiting the Hilbert irreducibility theorem	
16:20–17:20	Li Cai (Capital Normal University)
On the Archimedean Arithmetic smooth matching	

Saturday 23rd

Harder-Narasimhan stratifcation in p-adic Hodge theory

Jilong Tong

Capital Normal University

As a geometric variant of filtered isocrystals, we consider the notion of isocrystals equipped with a lattice. We show that there is a natural Harder-Narasimhan formalism for such objects, and the resulting Harder-Narasimhan filtration is compatible with tensor products. As an application, We will construct the Harder-Narasimhan stratification on the B_{dR}^+-Grassmannian. This generalizes the work of Dat-Orlik-Rapoport, Cornut-Peche Irissarry, Nguyen-Viehmann and Shen. This is a joint work in progress with Miaofen Chen.

Refined spectral halo for eigencurves and p-adic slopes of crystabelline representations

Bin Zhao

Capital Normal University

Coleman-Mazur-Buzzard-Kilford conjecture predicted that over the boundary of the weight space, the eigencurve is a disjoint union of rigid analytic spaces which are finite flat over the weight space. This conjecture has been proved by the work of Liu-Wan-Xiao and Diao-Yao. In this talk, I will report a joint work in progress with Yongquan Hu and Liang Xiao on a refinement of this conjecture and how it can be used to determine the p-adic slopes of all the crystabelline lifts of a reducible (local) mod p Galois representation. I will explain how to formulate the question under the context of p-adic local Langlands correspondence. The new ingredient is a universal principal series type theory that interpolates classical principal series types.

Hasse principle for odd genus hyperelliptic curves

Yongqi Liang

University of Science and Technology of China

Scharaschkin and Skorobogatov conjectured that the Brauer-Manin obstruction is the only obstruction to the Hasse principle for rational points on smooth projective curves defined over number fields. For any given number field and any given odd integer g, we explicitly construct infinitely many curves of genus g violating the Hasse principle explained by the Brauer-Manin obstruction. Our construction gives examples to support the conjecture. This is a joint work in progress with my PhD student Kai Huang.

Some new descent methods to weak approximation

Yang Cao

University of Science and Technology of China

The classical descent method, introduced by Colliot-Thélène and Sansuc, uses universal torsors to study the Hasse principle and weak approximation of algebraic varieties. Essentially, universal torsors are used to simplify the geometric structure (for example, its cohomology) of varieties. In this talk, I will try to find some new descent methods by using splitting varieties, which can simplify the geometric structure and also can study the weak approximation. I will only provide some abstract constructions, and hope it can apply to some examples in the future.

Strong approximation of the singular Del Pezzo surface

Dasheng Wei

Chinese Academy of Sciences

The rational points of singular Del Pezzo surface of degree ≥ 3 had widely studied by Skolen, Coray and Tsfasman. In this talk, we will give the strong approximation property of the smooth locus of singular Del Pezzo surface of degree ≥ 4. It is a joint work with Yi Zhu.

Sunday 24th

Strong approximation and integral quadratic forms over affine curves

Yong Hu

Southern University of Science and Technology

In the classical arithmetic theory of quadratic forms over global fields, strong approximation and the Hasse principle play a very important role. In this talk, we discuss extensions of some results in this direction to function fields of curves defined over more general fields. In particular, we give examples where strong approximation and the Hasse principle for integral quadratic forms hold, and examples where they do not hold. This is based on a joint work in progress with Jing Liu and Yisheng Tian.

Geometric Springer theorem for quadratic bundles over smooth affine curves under finite coverings of odd degree

Fei Xu

Capital Normal University

The classical Springer theorem says a quadratic form f is represented by another g over a field k if and only if f is represented by g over a finite extension of odd degree. Arithmetic version of this kind result is also true for indefinite integral quadratic forms. In this talk. we will explain the geometric version of this kind results. More precisely, we show that a quadratic bundle F can be imbedded into another quadratic bundle G over a smooth curve C if and only if the pullback of F can be imbedded into the pullback of G over a finite covering of odd degree by assuming G is isotropic over generic point of C. This is a joint work in progress with Jing Liu.

Local-global principle for zero cycles over global function fields

Zhiyu Tian

Peking University

A conjecture due to Colliot-Thélène describes the Chow group of zero cycles on a smooth projective variety defined over a global field in terms of the corresponding group over local fields and a Brauer-Manin obstruction. I will report on some recent work on this problem for geometrically rational surfaces and some higher dimensional generalizations over global function fields. The approach is mostly geometric.

Bounded Generation: a diophantine approximation approach

Jinbo Ren

Xiamen University

An abstract group is said to have the **bounded generation** property (BG) if it can be written as a product of finitely many cyclic subgroups. Being a purely combinatorial notion, bounded generation has close relation with many group theoretical problems including semisimple rigidity, Kazhdan's property (T) and Serre's congruence subgroup problem. This talk is devoted to explain how to use the Laurent's theorem in Diophantine approximation to prove that an infinite S-arithmetic subgroup of an anisotropic linear algebraic group G over a number field K **never** has (BG). If time allows, I will also introduce our newly obtained asymptotic formula for counting the elements of a (BG) set inside $GL_n(K)$ (K is a number field) when ordered by heights, together with some applications of this formula. The novelty of this project relies on the deep subspace theorem by Schlickewei-Schmidt as well as the theory of generic elements by Prasad-Rapinchuk. This is joint work with Corvaja, Demeio, Rapinchuk and Zannier.

Revisiting the Hilbert irreducibility theorem

Zhizhong Huang

Chinese Academy of Sciences

We say that a variety X over a field K is Hilbertian if $X(K)$ is not a thin set. Roughly speaking, $X(K)$ cannot be covered by the image of K-points from any finite collection of generically finite dominant morphisms of degree > 1. This notion originates from Hilbert's irreducibility theorem for projective spaces, and is weaker than the weak-weak approximation property. A conjecture of Corvaja and Zannier predicts that every algebraically simply connected smooth projective variety over a finitely generated field of characteristic 0 is Hilbertian. We report progress on Kummer varieties associated with jacobians of hyperelliptic curves, and on symmetric products of varieties. This is joint work in progress with Damián Gvirtz-Chen (Glasgow).

On the Archimedean Arithmetic smooth matching

Li Cai

Capital Normal University

We will firstly talk about the relative trace formula approach to the Gross-Zagier formula, especially the Archimedean arithmetic smooth matching. Then we discuss a general problem reducing the semi-global comparison to the local one. The talk is based on two joint works: one with Ye Tian, Xinyi Yuan and Wei Zhang, and the other one with Ye Tian.